ESC194 Unit 6.2

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Abstract

1 6.2

Identities:

$$ln(x) = \int_1^x \frac{dt}{t}$$

for

$$x > 0$$

$$\frac{d}{dx} ln\left|x\right|$$

for

$$x \neq 0$$

$$\int \frac{dx}{x} = ln\left|x\right| + c$$

$$\frac{d}{dx} ln\left|u\right| = \frac{1}{u}\frac{du}{dx} = \int \frac{du}{u} = ln\left|u\right| + c$$

Example:

$$\int \frac{6x+5}{3x^2+5x-1} dx = ln\left|3x^2+5x-1\right| + c$$

Example:

$$\int \frac{(ln(x))^2}{x} dx$$

let $u = lnx \therefore du = \frac{dx}{x}$

$$\int u^2 du = \frac{u^3}{3} + c = ...$$

Example:

$$\int tanx dx = \int \frac{sinx}{cosx}$$

let $u = cosx \rightarrow du = -sinxdx$

$$= ln\left|secx\right| + c$$

Example:

$$\int secx dx = \int ln\left|secx + tanx\right| + c$$

say:

$$g_1(x) \cdot g_2(x) \cdot g_3(x) ... g_n(x)$$

$$\rightarrow ln\left|g(x)\right| = ln\left|g_1(x)\right| + ln\left|g_2(x)\right| + ln\left|g_n(x)\right|$$

Apply differentiation to each individual term:

$$\frac{g'(x)}{g(x)} = \frac{g_1(x)}{g_1(x)} + \frac{g_2'(x)}{g_2(x)} + \frac{g_n'(x)}{g_n(x)}$$

or:

$$g'(x) = g(x)[\frac{g_1'}{g_2} + \frac{g_2'}{g_2}....]$$

This is called **Logarithmic Differentiation.**
Example:

$$g(x) = \frac{x^4(x-1)}{(x+2)(x^2+1)}$$

$$g'(x) = \frac{x^4(x-1)}{(x+2)(x^2+1)} \cdot [\frac{4x^3}{x^4} + \frac{1}{x-2} - \frac{1}{x+2} - \frac{2x}{x^2+1}]$$

6.3 The Natural Exponential Function

$$ln(e^{\frac{p}{q}}) = \frac{p}{q}$$

For example there must be some number q such that:

$$ln(q) = \pi$$

We give this number q the value of, $q = e^{\pi}$

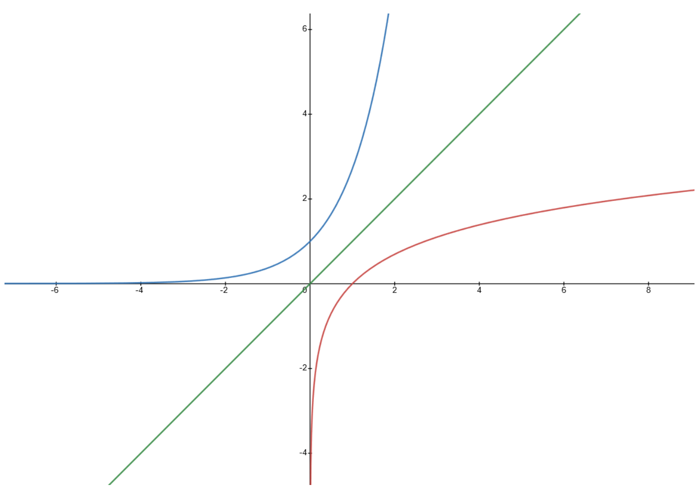
Definition: If z is an irrational, then e^z is a unique number such that $ln(e^z) = z$

Definition: $\exp(x) = e^x$ **Exponential Function**

Properties of e^x

1) $ln(e^x) = x$ for all real numbers

2) Due to inverse relationship between $ln(x)$ and e^x, they are mirrored across the x = y line:



3) $e^x > 0$

4)$e^x = 1$ when$x = 0$ and $ln(x) = 0$ when $x = 1$

5)$\lim_{x \to -\infty} e^x = 0$

6)$e^{lnx} = x$

7)$e^{a+b} = e^a \cdot e^b$
$ln(e^a \cdot e^b) = ln(e^a) + ln(e^b) = a + b = lne^{a+b}$

9)
$$\frac{d}{dx}e^x = e^x$$
$$lne^x = x-> \frac{d}{dx}lne^x = \frac{1}{e^x} \cdot (e^x)' = 1$$
$$\therefore (e^x)' = e^x$$

10)
$$\frac{d}{dx}e^{u(x)} = e^{u(x)}\frac{du}{dx}$$

Example:
$$\frac{d}{dx}e^{kx} = ke^{kx}$$

Example:
$$f(x) = x^4e^{-x}$$

Domain: all x, $f \geq 0$ all x
$$f'(x) = 4x^3e^{-x} + x^4(-1)e^{-x} = x^3e^{-x}(x+4)$$
$$f' = 0 \to x = \{0, 4\}$$
$$f' < 0 \to x > 4)$$
$$f' > 0 \to 0 < x < 4$$
$$f' < 0 \to x < 0$$

Therefore: $f(0) = 0 \to$ local minimum
$$f'' = 12x^2e^{-x} - 4x^3e^{-x} - 4x^3e^{-x} + x^4e^{-x}$$

$$= x^2e^{-x}(x^2 - 8x + 12) = x^2e^{-x}(x-6)(x-2)$$

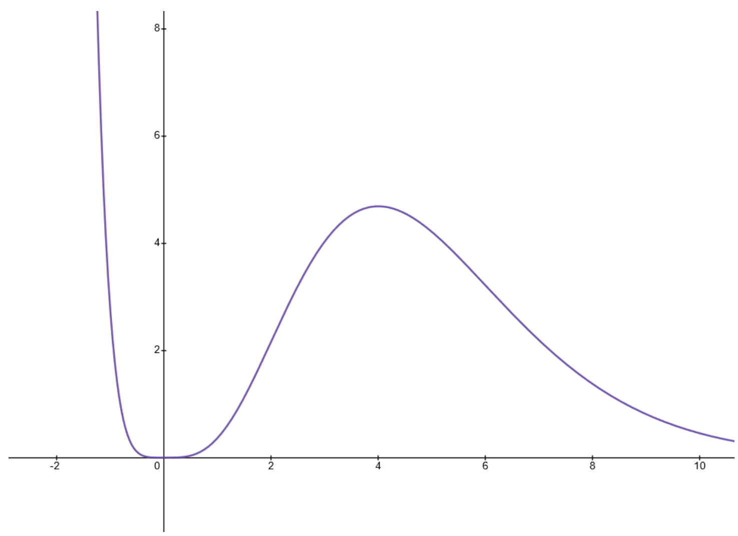
$$f'' = x \to x = \{2, 6, 0\}$$

$$f'' > 0 \to x > 6 \therefore concup$$

$$f'' < 0 \to 2 < x < 6 \therefore concdown$$

$$f'' > 0 \to x < 2 \therefore concup$$

$x = 6$ and $x = 2$ are inflection points



$$c(t) = At^pe^{-kt}$$

$$t > 0$$

Properties again: 11)

$$\int^x dx = e^x + c$$

12)

$$\int e^{g(x)}g'(x)dx = e^{g(x)} + c$$

Example:

$$I = \int \frac{xe^{ax^2}}{e^{ax^2}+1}dx$$

let $u = e^{ax^2} \therefore du = 2axe^{ax^2}$

$$I = \frac{1}{2a}\int \frac{du}{u} = \frac{1}{2a}ln\left|e^{ax^2}+1\right| + c$$

$$= \frac{1}{2a}ln(e^{ax^2}+1) + c$$

Example:

$$\int_0^{\sqrt{2ln3}} xe^{\frac{-x^2}{2}}dx$$

$$u = -\frac{1}{2}x^2 \therefore du = -xdx$$

$$x = 0 \rightarrow u = 0$$

$$x = \sqrt{2ln3} \rightarrow u = -ln3$$

$$-\int_0^{ln3} e^u du$$

$$= 1 - e^{ln3} = 1 - \frac{1}{3} = \frac{2}{3}$$

Example: Show:

$$e^x > 1 + x$$

$$e^x = 1 + \int_0^x e^t dt$$

Evaluate to show:

$$1 + e^x - e^0 = e^x$$

$e^x > 1$ for all x ¿ 0

$$e^0 = 1\& \frac{d}{dx}e^x = e^x > 0 \therefore increaseing$$

$$1 + \int_0^x e^t dt > 1 + \int_0^x dt$$

$$= 1 + x$$

2 HI GUYS :)

$$u = qt\pi$$